A POWERFUL TOOL FOR THE SYNTHESIS OF PROTOTYPE FILTERS WITH ARBITRARY TOPOLOGY

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ABSTRACT — **This paper describes a CAD tool for synthesizing prototype filters with arbitrary topology, with cross-couplings starting also from source and (or) from load; the generalized Chebycheff characteristic is assumed, with asymmetric transmission zeros, both pure imaginary or complex. The proposed method employs the coupling matrix of a generic prototype, obtained through well-established procedures; then the coupling matrix of the desired topology is determined with a procedure based on multiple matrix rotations (similarity transforms) and numerical optimization (the sequence and angles of the rotations are automatically evaluated in the procedure and their knowledge is not a priori required). The novel CAD tool has been tested with some coupling schemes recently proposed, which don't require diagonal couplings for realizing asymmetric transmission zeros.**

I. INTRODUCTION

For satisfying the severe selectivity requirements in the modern diplexers for base stations of mobile communication systems, filter characteristic with several frequency asymmetric transmission zeros (both imaginary or complex) need to be realized. Then the interest in the synthesis methods of prototype filters with asymmetric frequency response is greatly increased in these last years; starting from some "keystone" works concerning both the problem of approximation and the network synthesis [1,3], many proposals have appeared in the literature with the general objectives of synthesizing the final network more easily and to find out more and more convenient topologies [4, 7]. It should be however observed that, for what concerns the prototype network for a given pole-zero pattern, does not yet exist a true synthesis method that allow to obtain an arbitrary topology (for which the feasibility has been assessed). In this work we have developed a general CAD tool that allow the "true" synthesis of a very general prototype network, exhibiting the generalized Chebycheff response, with a maximum of N-2 arbitrary placed transmission zeros (with N number of poles), with cross couplings starting also from source and/or load. The method employed is based on the matrix rotation (similarity transform), applied to the coupling matrix of the canonical folded prototype (which is always synthesisable for a generalized Chebycheff characteristic [2]); using a suitable optimisation procedure, the main limitation of this methodology has been overcame, i.e. the determination of the unknown rotations sequence and the corresponding angles. For demonstrating the proposed tool, some novel prototype topologies recently introduced (which don't make use of diagonal cross couplings) have been synthesized using multiple couplings from source and load to reduce the overall number of resonators; the results obtained are illustrated in the paper.

II. SYNTHESIS METHODOLOGY

It is here assumed that the prototype exhibits the generalized Chebycheff response, with a maximum number of transmission zeros equal to *N-2* (being *N* the order of the prototype); once N, the passband return loss *RL* and the transmission zeros f_{zk} are given, various methods may be found in the literature which allow to compute the characteristic polynomials determining the synthesis of this prototype (they are generally indicated as *E(s), F(s), P(s)* [2,3,8]; their zeros represent respectively the poles, the reflection zeros and the transmission zeros). From these polynomials, it is also possible to obtain automatically the elements of a canonical folded prototype following the procedure presented by Cameron in [2]; in alternative, the method proposed by Atia [1,5] allows to compute, through an ortho-normalization technique, a prototype with all the possible couplings between the resonators. We then assume as a starting point to have a general coupling matrix describing the prototype so evaluated which can be represented in the following form:

$$\mathbf{M}_P = \begin{matrix} 0 & J_{0,1} & 0 & . & 0 \\ J_{0,1} & b_1 & J_{1,2} & . & 0 \\ . & . & . & . & . \\ 0 & . & J_{N-1,N} & b_N & J_{N,N+1} \\ 0 & . & 0 & J_{N,N+1} & 0 \end{matrix} \qquad (1)$$

Note that $\boldsymbol{M_P}$ has order *N+2* because generator and load are included in the matrix topology; however, having

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assumed that the maximum number of transmission zeros are *N-2*, in the first (last) raw (column) there are only the coupling $J_{01}=\sqrt{1/R_S}$, $J_{N,N+1}=\sqrt{1/R_L}$, where R_S and R_L are the source and load resistances as obtained from the synthesis of the prototype (note that they are in general different from unity).

We now describe a method which, starting from the matrix M_P, allows to derive a new coupling matrix $\mathbf{M}$ having a very arbitrary topology, but presenting the same frequency response associated with M_P (obviously, the new topology must allow the synthesis of the imposed response, with the specified transmission zeros).

As well known, a possible method for deriving the matrix $\mathbf{M}$ is the matrix rotation method. This method has been widely employed for the synthesis of cross-coupled prototype filters [1,3,5]; its main drawback is that the suitable sequence of matrix rotations required for given starting and ending (transformed) topologies it is not known a priori. Here we propose an implementation of this method that does not require the knowledge of the optimum sequence of rotations, which is determined through a suitable optimization procedure. It is known [3] that a similarity transform of the matrix M_P of order n (equal to $N+2$) is defined by $\mathbf{R}_{ij}(\vartheta)\cdot\mathbf{M}_P\cdot\mathbf{R}'_{ij}(\vartheta)$, where $\mathbf{R}_{ij}(\vartheta)$ is the rotation matrix of order n, pivot (i,j) and angle ϑ, defined as follows:

$$
\begin{aligned}
&R_{ij}(i,i)=R_{ij}(j,j)=\cos(\vartheta),\\
&R_{ij}(i,j)=-R_{ij}(j,i)=\sin(\vartheta)\\
&R_{ij}(k,k)\big|_{k\neq i,j}=1, \qquad\qquad (i<j)\neq 1,N \qquad (2)\\
&R_{ij}(k,i)\big|_{k\neq i,j}=0,\ R_{ij}(j,k)\big|_{k\neq i,j}=0.
\end{aligned}
$$

The conservation of the transfer function is also true for subsequent applications of the above transformation; so we assume that the transformed matrix $\mathbf{M}$ can be expressed as:

$$
\begin{aligned}
\mathbf{M}&=\left(\mathbf{R}_{23}\cdot\mathbf{R}_{24}\cdot\cdot\mathbf{R}_{N-2,N-1}\right)\cdot\mathbf{M}_P\cdot\left(\mathbf{R}'_{N-2,N-1}\cdot\mathbf{R}'_{N-3,N-1}\cdot\cdot\mathbf{R}'_{23}\right)=\\
&=\mathbf{S}(\vartheta_1,\vartheta_2,..\vartheta_M)\cdot\mathbf{M}_P\cdot\mathbf{S}'(\vartheta_1,\vartheta_2,..\vartheta_M) \qquad (3)
\end{aligned}
$$

where m is the overall number of distinct pivots (i,j), for a given order n; m may be considered as the maximum number of independent rotations, and it is given by: $m=(n^2-5n+6)/2$. It can be observed that the topology of the transformed matrix $\mathbf{M}$ is determined by the set of rotation angles $(\vartheta_1, \vartheta_2,..\vartheta_m)$; in fact, assuming that the desired topology of the transformed matrix is compatible with the original frequency response (associated to M_P), the required values of the rotation angles can be found numerically by imposing to zero the elements of $\mathbf{M}$ corresponding to the cross-couplings not included in the new topology. In other words, the set of rotations angles $(\vartheta_1, \vartheta_2,..\vartheta_m)$ can be found by solving the following system of non-linear equations:

$$\mathbf{M}_{k,l}(\vartheta_1,\vartheta_2,\ldots\vartheta_M)=0 \qquad (4)$$

where k and l refer to all the elements of $\mathbf{M}$ which must vanish (only the elements above the main diagonal are considered being the matrix symmetric).

The numerical solution of the above system may be performed through the minimization of a non-linear cost function U defined as:

$$U=\sum_{k,l}\left|\mathbf{M}_{kl}(\vartheta_1,\vartheta_2,\ldots\vartheta_M)\right|^2 \qquad (5)$$

In the practical implementation of the minimization procedure, the Gauss-Newton method has been used, because it allows a fast and accurate solution and it is little sensitive to the starting point.

III. Examples of Prototypes Synthesis

For illustrating the capabilities of the synthesis tool here introduced, two innovative topologies for the prototype will be considered, which don't make use of diagonal cross-couplings.

A. Test Prototype A

Here the box section proposed in [8] is assumed to be realized with source (or load) and 3 resonators suitably coupled. As known, a box section allows placing one asymmetric zero in the response without using diagonal couplings. With two of these sections, a prototype having the following specifications will be synthesized:

$$N=6,\ f_z=[\,j1.2,\ j1.5],\ RL=25\text{ dB}$$

where f_z are the transmission zeros. From these specifications, the canonical folded prototype has been first synthesized [2], giving the following coupling matrix:

$\mathbf{M}_P$ =

0	1.1038	0	0	0	0	0	0
1.1038	0.0506	0.9424	0	0	0	0	0
0	0.9424	0.0645	0.3339	0.5244	0.2102	0	0
0	0	0.3339	-0.9146	0.2166	0	0	0
0	0	0.5244	0.2166	-0.2342	0.6217	0	0
0	0	0.2102	0	.6217	0.0645	0.9424	0
0	0	0	0	0	0.9424	0.0506	1.1038
0	0	0	0	0	0	1.1038	0

By applying the matrix rotation optimization here introduced, the topology constituted by two cascaded box sections illustrated in fig. 1 (including source and load) is obtained; the coupling matrix **M** of the synthesized topology is also reported in the following. The frequency response of this prototype is reported in fig. 2.

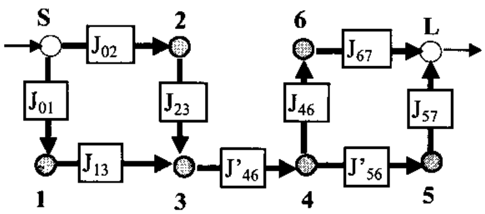

Fig. 1. Topology of the Prototype A. Each node represents a unit capacitance in parallel with a frequency invariant susceptance b_i; J_{ij} are the coupling coefficients between resonators i and j. Load (L) and generator (S) are equal to unit.

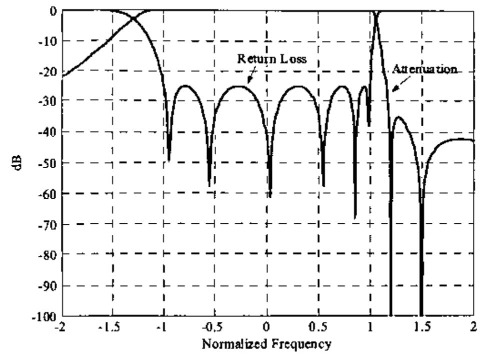

Fig. 2. Frequency response of the Prototype A.

	0	**0.9662**	**0.5337**	**0**	**0**	**0**	**0**	**0**
	0.9662	**0.4235**	**0**	**0.7646**	**0**	**0**	**0**	**0**
	0.5337	**0**	**-1.1716**	**-0.0242**	**0**	**0**	**0**	**0**
M =	**0**	**0.7646**	**-0.0242**	**0.1715**	**-0.6173**	**0**	**0**	**0**
	0	**0**	**0**	**-0.6173**	**0.1638**	**-0.1698**	**-0.6790**	**0**
	0	**0**	**0**	**0**	**-0.1698**	**-1.1298**	**0**	**-0.6311**
	0	**0**	**0**	**0**	**-0.6790**	**0**	**0.6239**	**0.9056**
	0	**0**	**0**	**0**	**0**	**-0.6311**	**0.9056**	**0**

B. Test Prototype B

The configuration considered is the so-called "cul-de-sac" that allows to place up to N-3 complex zeros without diagonal cross-couplings [8]. In this case the following specifications are imposed:

N=6, f_z=[j1.2, j1.5, ±0.5-j0.15], RL=25 dB

Again the synthesis of the initial coupling matrix M_P has been obtained thorugh the canonical folded prototype, giving the following result:

	0	**1.1327**	**0**	**0**	**0**	**0**	**0**	**0**
	1.1327	**0.1152**	**0.9075**	**0**	**0**	**0.3857**	**0.2687**	**0**
	0	**0.9075**	**-0.0499**	**0.4326**	**0.2477**	**0.1096**	**0**	**0**
M_P =	**0**	**0**	**0.4326**	**-0.7957**	**0.2609**	**0**	**0**	**0**
	0	**0**	**0.2477**	**0.2609**	**-0.1819**	**0.6148**	**0**	**0**
	0	**0.3857**	**0.1096**	**0**	**0.6148**	**0.0433**	**0.9861**	**0**
	0	**0.2687**	**0**	**0**	**0**	**0.9861**	**0.1152**	**1.1327**
	0	**0**	**0**	**0**	**0**	**0**	**1.1327**	**0**

Fig. 3 illustrates the topology of the prototype B and the corresponding frequency response; the matrix M computed with the novel procedure is also reported in the following.

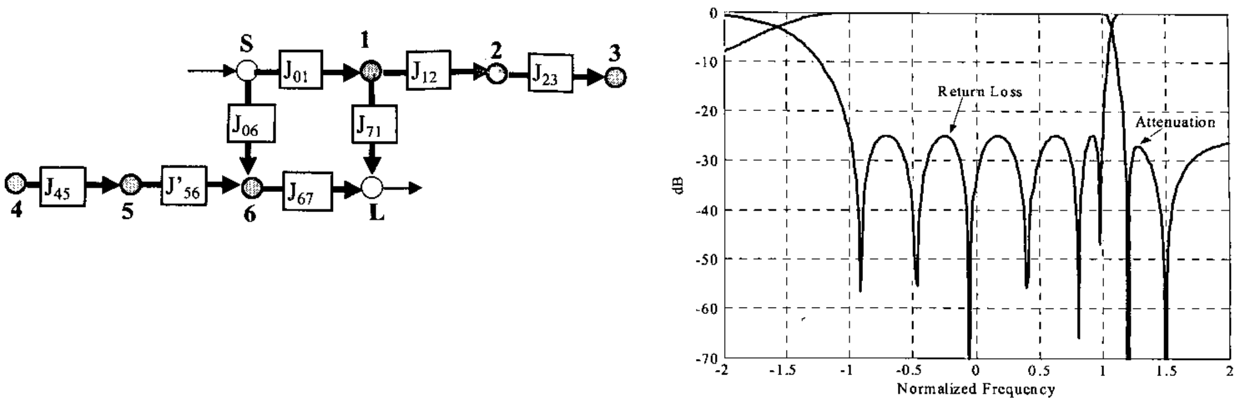

Fig. 3. Topology of the Prototype B and its frequency response.

M =

0	**0.8010**	**0**	**0**	**0**	**0**	**0.8010**	**0**
0.8010	**-0.1534**	**0.7694**	**0**	**0**	**0**	**0**	**-0.8010**
0	**0.7694**	**-0.1224**	**0.3844**	**0**	**0**	**0**	**0**
0	**0**	**0.3844**	**-0.8916**	**0**	**0**	**0**	**0**
0	**0**	**0**	**0**	**-0.0860**	**0.6919**	**0**	**0**
0	**0**	**0**	**0**	**0.6919**	**0.1158**	**1.1631**	**0**
0.8010	**0**	**0**	**0**	**0**	**1.1631**	**0.3839**	**0.8010**
0	**-0.8010**	**0**	**0**	**0**	**0**	**0.8010**	**0**

IV. Conclusion

A novel tool CAD for the true synthesis of prototype filters with arbitrary topology and generalized Chebycheff response has been introduced. It has been applied for synthesizing some innovative couplings schemes including source and load, which don't make use of diagonal cross couplings for realizing asymmetric transmission zeros.

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